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Valuation of Portfolio Credit Default Swaptions

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We discuss the valuation of options on portfolio credit default swaps with a focus on standardized contracts referencing for example the CDX or TRAC-X entities. First, we describe the portfolio swap and swaption contracts. Next, we argue that Black formulas, the standard formulas for pricing single-name default swaptions, are inappropriate for pricing portfolio default swaptions. Finally, we present a simple, easy to implement, alternative model that prices portfolio swaptions using the credit curves of the reference entities and a single volatility parameter.¹

1. INTRODUCTION

Investors are increasingly finding portfolio credit default swaps (also called portfolio CDS, portfolio default swaps or, in this report, simply portfolio swaps) useful for gaining exposure to market wide credit spreads. Options on portfolio default swaps (portfolio credit default swaptions, portfolio default swaptions, or simply portfolio swaptions) allow investors to leverage this exposure and provide a tool for gaining exposure to market wide credit spread volatility. Standardized portfolio default swaps referencing the CDX.NA.IG (in this report simply CDX) and TRAC-X NA (in this report simply TRAC-X) entities are today trading with bid-offer spreads as low as 1-2bp. Recently, CDX and TRAC-X swaptions have seen increasing trading volume as well. Lehman Brothers is a market maker in CDX and TRAC-X portfolio swaps and swaptions.

The purpose of this research report is to introduce the CDX and TRAC-X portfolio swaptions and present a simple model for their pricing and risk management. Such a model already exists for single-name default swaptions in the form of a modification of Black's formulas for interest rate swaptions. These Black formulas for default swaptions provide the values of swaptions that knock out if the reference entity defaults before swaption maturity. To price an option to buy protection that does not knock out, it is therefore necessary to add to the Black formula price the value of protection until swaption maturity. Portfolio swaptions do not knock out. It has therefore been suggested to price these as single-name non-knockout default swaptions using a credit curve representing the average credit worthiness of the reference entities. However, such an approach will tend to overvalue high strike portfolio swaptions compared with a model that more explicitly models the underlying cashflow.

The easiest way to see that Black formulas give mispricing is by examining the pricing of a deep out-of-the-money payer swaption (this is an option to buy portfolio protection at a very high strike spread). If it is priced as suggested above, its value will be close to the value of protection until swaption maturity. This is incorrect; the price should approach zero as the strike increases. This is because, when the strike is very high, the payer will not be exercised even if a reference entity has defaulted unless the portfolio spread on the non-defaulted entities has widened sufficiently.

We propose to directly model the terminal value of the swaption using a single state variable which we call the default-adjusted forward portfolio spread. As long as no defaults have occurred, this spread can reasonably be called the forward portfolio spread since it is the

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strike for which the payer and receiver portfolio swaptions are equal in value. If no defaults have occurred at swaption maturity, the spread is simply the portfolio spread itself. As we explain in detail below, we propose to incorporate defaults directly into the spread and thereby avoid modeling the number of defaults which would otherwise be required to explicitly model the terminal value of a portfolio swaption.

In section 2, we describe the CDX and TRAC-X portfolio swap and swaption contracts. In section 3, we explain why the Black formulas for default swaptions should not, as suggested by others, be used to price portfolio swaptions. In section 4, we present our alternative valuation model. Section 5 concludes.

2. THE PORTFOLIO SWAP AND SWAPTION CONTRACTS

To value a portfolio swaption it is necessary to understand the details of the swaption contract. The presentation is focused on CDX but, as explained below, TRAC-X works almost identically.

2.1. The CDX swap contract

A buyer of CDX protection is buying credit protection on 125 fixed reference entities. If the notional of the CDX swap is 125 million, say, then buying CDX protection has the same economic effects as buying protection on each reference entity through 125 single-name No-R² CDS, each with a notional of 1 million. A buyer of CDX protection is obligated to pay the same premium (called the *fixed rate*) on all the 125 hypothetical underlying CDS. When a default occurs, the CDX protection buyer must physically settle to receive the protection payment for the defaulted entity. After settlement, the relevant hypothetical underlying CDS is eliminated from the CDX swap, which then has a notional that is decreased by 1/125 of the original notional and one less reference entity. The fixed rate remains unchanged. The payment dates are the standard CDS dates (20th of March, June, September and December).

Today there are two CDX swaps with 5- and 10-year maturities. Currently the fixed rate is 60bp for the 5-year contract and 70bp for the 10-year contract. On the 20th of March and September each year (the *roll dates*), new CDX swaps will become *on-the-run* to ensure that the reference entities represent the aggregate market and that the on-the-run contracts have 5- and 10-year maturities³. In addition to reference entities and maturities, the fixed rates will also be changed⁴. Changes to new on-the-run CDX swaps have no influence on the existing contracts. The maturity, the fixed rate, and the reference entities are all fixed throughout the life of specific CDX swaps and swaptions.

Reference entities may also be eliminated from the on-the-run CDX swaps between roll dates, for example, if an entity defaults. Eliminated entities will not be replaced between roll dates. If an entity is eliminated, the relevant hypothetical underlying CDS can be split off from existing CDX swap contracts to ensure that they remain on-the-run. This ensures liquidity of the CDX swap contract after a default has been settled.

² No-R means 'no restructuring' and refers to the fact that only default (bankruptcy and failure to pay) can trigger the CDX contract. In the US, CDS mainly trade under the Mod-R (modified restructuring) clause which includes restructurings as a credit event. For details see O'Kane, Pedersen and Turnbull (2003).

³ The reference entities in the new on-the-run swap are chosen by voting among CDX market makers. The voting system is fundamentally different from the rules-based methodology used to determine the constituents of the Lehman Brothers Credit Default Swap Index. See Berd et al. (2003) for more details.

⁴ After publication of the new reference entities, CDX market makers will submit 5- and 10-year quotes, the medians of which will become the new fixed rates.

The value of a CDX swap is driven by the CDS spreads on the 125 reference entities. When the spreads are high compared with the fixed rate, the swap has a positive value to the protection buyer who therefore pays a *CDX price* to the protection seller at contract initiation (when the swap price is negative its absolute value is paid by the protection seller to the protection buyer). The swap price is quoted in the form of a *CDX spread* which can be readily converted into the price.

The conversion from spread to price can be done via the CDSW calculator on Bloomberg. The calculation is to discount fixed payments equal to the CDX spread minus the fixed rate, as when calculating the mark-to-market on a CDS⁵. The conversion can be written as:

$$P = PV01 \cdot (S - FR)$$

where FR is the fixed rate, S is the quoted CDX spread, P is the corresponding CDX price, and PV01 is the risky PV01 from today to swap maturity, ie, the value of receiving 1bp on the CDX payment dates until maturity of the swap or default, whichever occurs first.

The market standard for quoting CDX spreads assumes that the PV01 is calculated using discount factors that have been calibrated to fit a flat CDS curve with spreads equal to the CDX spread. The calibration uses a recovery-given-default of 40% and default-free interest rates taken from the current Libor curve.

The intrinsic CDX spread

The intrinsic CDX spread is the spread quote that converts into the price of buying the CDX equivalent credit protection through 125 single-name CDS contracts, each with a contractual spread equal to the CDX fixed rate. The intrinsic spread depends on the shape of the individual credit curves, but will be close to the average CDS spread of the appropriate maturity across the CDX reference entities.

Determination of the intrinsic spread is made more difficult by the fact that restructuring is not included as a credit event in the CDX swap contract. No-R CDS spreads are rarely available, but in the US, dealers tend to quote No-R spreads around 5% lower than Mod-R spreads. Intrinsic CDX spreads are usually calculated from Mod-R spreads that have been discounted 5% (see O’Kane, Pedersen and Turnbull (2003)).

Aside from possible error introduced when determining No-R spreads, a CDX spread may differ from its intrinsic value because of specific shorter term demand-supply conditions in CDX and/or CDS markets. Because of the No-R feature, and because of bid-offer spreads in CDS markets, it is generally not feasible to arbitrage the differences between quoted and intrinsic CDX spreads.

Currently, intrinsic CDX spreads tend to be higher than quoted CDX spreads implying that it is cheaper to buy protection through CDX than through the underlying single-name CDS. Part of the difference may be a result of the higher liquidity of CDX compared to CDS. In other words, sellers of CDX protection can be seen as demanding a lower liquidity premium than sellers of single-name CDS protection.

⁵ See O’Kane and Turnbull (2003) for details on how to calculate the mark-to-market of a CDS.

2.2. The CDX swaption contract

A CDX swaption references a specific underlying CDX swap with a specific maturity. Usually the CDX swap was on-the-run when the option was traded. Aside from the underlying CDX swap, the CDX swaption is specified by a swaption maturity, a strike spread, and a swaption type. The standard portfolio swaptions are European. The swaption type is either *payer* or *receiver*. A payer gives the right to become a protection buyer in the underlying CDX swap at the strike spread. A receiver gives the right to become a protection seller.

If the swaption is exercised, the strike spread is converted into an *exercise price* (also called the settlement payment) using the same calculation as when converting a CDX spread quote into a CDX price. For example, if the strike spread is K , then the exercise price is:

$$P(K) = \gamma(K)(K - FR)$$

where FR is the fixed rate in the underlying CDX swap contract, and $\gamma(K)$ is the risky PV01 calculated at the exercise date using the same assumptions as if converting a CDX spread quote of K into its corresponding CDX price. That is, $\gamma(K)$ is the risky PV01 calculated from a credit curve that has been calibrated to a flat CDS curve with spreads equal to the strike spread, K , using a recovery-given-default of 40% and the Libor curve at the exercise date.

Consider an example. On November 6, 2003, a CDX payer swaption on the 5-year CDX swap with a notional of \$100 million was traded. The swaption maturity is March 22, 2004, the strike spread is $K = 55\text{bp}$, and the fixed rate is $FR = 60\text{bp}$. If we assume that the Libor curve on March 22, 2004 is equal to the forward Libor curve for that date observed on November 6, 2003, then we can find the exercise price. Under this Libor curve, the risky PV01 is $\gamma(K) = 0.0453$ (\$ per \$100 notional). The exercise price is then $0.0453 \cdot (55 - 60) = -0.227$, so if the swaption is exercised, the protection seller must pay \$227 thousand to the protection buyer. This cash transfer of \$227 thousand is independent of the number of defaults that may have occurred before swaption maturity and the CDX spread at swaption maturity. The only uncertainty about the amount arises from uncertainty about the Libor curve at the exercise date.

When the swaption is exercised, the protection buyer has in effect bought protection on all reference entities, including those that may have defaulted before swaption maturity. The protection buyer can then immediately settle for protection payment on any defaulted entities. The terminal value of the swaption therefore depends on the recoveries on defaulted entities and the mark-to-market on the CDX swap with the defaulted entities eliminated.

Depending on the maturity of the swaption and the time from the option trade date to the following roll date, the CDX swap with the defaulted entities eliminated may or may not be on-the-run at option maturity. However, given the relatively short swaption maturities usually seen (most swaptions trade with maturities of six months or less), this CDX swap should be liquid throughout the life of the swaption.

Let us extend the example above to determine the terminal value of the swaption. Assume that one reference entity defaulted before March 22, 2004, and the cheapest deliverable obligation issued by the defaulted entity trades at \$45 (per \$100 notional). Also assume that the quoted market CDX spread on March 22, 2004, on the CDX swap with the defaulted entity eliminated is 75bp. Finally, assume as above that the Libor curve on March 22, 2004, is the forward Libor curve for that date observed on November 6, 2003. In this case, the payer swaption will be exercised and the swaption seller must make an initial \$227 thousand cash payment to the swaption buyer at exercise (see details above). After exercise, the swaption

buyer can deliver \$800 thousand of notional of the deliverable obligation mentioned above and receive a \$800 thousand cash payment. The cost of the deliverable is \$360 thousand. The last step in determining the terminal value of the swaption is to find the mark-to-market of the CDX swap with the defaulted entity eliminated. The notional of this CDX swap is \$99.2 million and the market value to the swaption buyer of this position turns out to be \$675 thousand (found as $0.0450 \cdot (75-60) \cdot \$99.2 \text{ million}/100$, given that the risky PV01 calculated from a flat curve of 75bp is 0.0450). Altogether the payer swaption has a total value of \$1.342 million ($227+800-360+675$). Notice that for this particular example, the effect of one default is about the same as a 10bp widening in the CDX spread on the non-defaulted entities.

2.3. The TRAC-X swap and swaption contracts

TRAC-X swap and swaption contracts work in the same way as the CDX swap and swaption contracts described above. The differences are the number of reference entities, the identity of the reference entities, the fixed rate, and the procedure for determining new reference entities. We are not aware of any other differences with pricing implications.

There are currently 100 TRAC-X reference entities the composition of which is scheduled for change every three months (around the standard CDS dates, ie, the 20th of March, June, September, and December). A change to new on-the-run swap contracts occurred in September 2003 when 34 entities were replaced. The fixed rate is currently 100bp in both the old and the new TRAC-X swap contracts. On November 6, 2003, TRAC-X was trading at 53 and a TRAC-X protection seller would have to make a significant upfront payment at contract initiation (more than \$2 million on a contract with a \$100 million notional). Currently, there are on-the-run TRAC-X swaps with 5- and 10-year maturities.

The remainder of this report uses generic language, and the issues apply to both CDX and TRAC-X swaptions. We use terminology such as portfolio swap, portfolio swaption and portfolio spread.

3. BLACK FORMULAS FOR DEFAULT SWAPTIONS

Black formulas for default swaptions are simple theoretically consistent formulas for valuing single-name default swaptions. The formulas give values of swaptions that knock out if a default occurs before swaption maturity. The knockout feature is not relevant for receiver swaptions (options to sell protection), as they will never be exercised after a default. For payer swaptions (options to buy protection), it is necessary to add to the Black formula price the value of protection from the swaption trade date to the swaption maturity.

To introduce notation, let $T > 0$ be the swaption maturity and let $T_M > T$ be the maturity of the underlying CDS. Let $PV01_t(T, T_M)$ be the value at time $t \leq T$ of a security that pays a 1bp annual flow starting at the first CDS date after time T and ending at T_M or default, whichever occurs first. Similarly, let $PVP_t(T, T_M)$ be the value at time t of a security that pays par minus the recovery-given-default at the time of default if default occurs between time T and time T_M . Given a Libor curve and an issuer CDS curve at time t , $PV01_t(T, T_M)$ and $PVP_t(T, T_M)$ can be found using the Jarrow-Turnbull credit pricing framework with a piecewise linear hazard rate. Furthermore, for $t \leq T$ let:

$$F_t(T, T_M) = \frac{PVP_t(T, T_M)}{PV01_t(T, T_M)}$$

$F_t(T, T_M)$ is the T-forward-starting spread on a CDS that matures at time T_M .

The Black formulas are based on the assumption that:

$$F_T(T, T_M) = F_0(T, T_M) \exp\left(-\frac{1}{2}\sigma^2 T + \sigma\sqrt{T}\varepsilon\right)$$

where ε is a standard normal random variable under the risk neutral measure, corresponding to $F_t(T, T_M)$ being lognormally distributed with volatility σ . The formulas are⁶:

$$PS_0^{KO} = PV01_0(T, T_M) \cdot (F_0(T, T_M) \cdot N(d_1) - K \cdot N(d_2))$$

$$RS_0 = PV01_0(T, T_M) \cdot (K \cdot N(-d_2) - F_0(T, T_M) \cdot N(-d_1))$$

$$d_1 = \frac{\log(F_0(T, T_M)/K) + \sigma^2 T/2}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

where K is the strike, PS_0^{KO} is the knockout payer value and RS_0 is the receiver value. The value of a payer that does not knock out is:

$$PS_0 = PS_0^{KO} + FEP_0(0, T)$$

where $FEP_0(0, T)$ (front-end protection) is the value at time t of a contract that pays par minus the recovery-given-default at swaption maturity, T , if default occurs between time 0 and time T . Determining the forward spread, the forward PV01, and the front-end protection, requires a CDS curve for the issuer. The shape of the curve is significant in the valuation.

It has been suggested that the above methodology can be used to value a portfolio swaption, using a CDS curve that represents the average creditworthiness of the reference entities. As an approximation we could, for example, for each maturity calculate the average spread across reference entities and shift the averages to ensure that the 5- and 10-year points match the quoted portfolio spreads. We could also adjust the averages in such a way that if all the reference entities had the adjusted average curve, then the intrinsic portfolio spreads would equal the quoted portfolio spreads.

Although this approach is appealing because of its simplicity, it is not theoretically sound. It is easy to see this by examining the pricing of payer swaptions as the strike increases. For a very high strike, the above approach will give a price close to FEP, the value of the front-end protection. When the strike is infinite, however, the price should be 0, since such a swaption should not be exercised even if most entities defaulted⁷.

If all reference entities had the same CDS curve, then the approach would correctly price the difference between the payer and the receiver (this will become clearer in the next section). From that perspective, the front-end protection has to be included in the payer price. However, this causes the decomposition into the payer and receiver prices to become skewed towards high prices for high strike swaptions.

When the reference entities do not have the same CDS curves the pricing of the difference between the payer and the receiver also breaks down. Using the average curve will tend to overvalue a long-payer short-receiver position compared with the approach we suggest in the

⁶ For more details on how to derive the formulas, see the section on modeling credit options in the Lehman Brothers Guide to Exotic Credit Derivatives and the references given there.

⁷ Theoretically the swaption could be exercised if all entities defaulted and had a combined recovery of less than 40%.

next section. This is mainly caused by the concavity in the value of the protection leg of a CDS when seen as a function of the CDS spread level.

4. A MODEL FOR PORTFOLIO DEFAULT SWAPTIONS

To explicitly model the terminal value of a portfolio swaption it is necessary to model both the recovery on defaulted entities and the spread on the portfolio swap with the defaulted entities eliminated. As a simplification, we suggest incorporating defaults directly into the portfolio spread to arrive at a *default-adjusted forward portfolio spread* that can be used as a single state variable to generate the terminal value of the swaption. Alternatively, the terminal value could be modeled directly, but it is not clear which distribution would be reasonable. On the other hand, we find it reasonable to use as a first approximation a lognormal distribution for the default-adjusted forward portfolio spread at swaption maturity.

4.1. The put-call parity

The first step in valuing a portfolio swaption is to identify and value the underlying. We refer to the underlying as the default-adjusted forward portfolio swap. It is different from a regular knockout forward portfolio swap because protection for defaults that occur before swaption maturity are paid at swaption maturity.

The default-adjusted forward portfolio swap can be viewed as a portfolio of non-knockout forward starting CDS on each reference entity. A non-knockout forward starting CDS is a combination of a regular knockout forward starting CDS and front-end protection from today to swaption maturity. The contractual spread in the CDS is the fixed rate in the portfolio swap, denoted FR. The value of such a forward starting CDS at time $t \leq T$ is:

$$V_t^i = PVP_t^i(T, T_M) - PV01_t^i(T, T_M) \cdot FR + FEP_t^i(0, T)$$

where an i superscript indicates that the values are for the i 'th reference entity. T is the swaption maturity and $T_M \geq T$ is the maturity of the CDS. The PVP and PV01 notation was explained in the previous section. $FEP_t^i(0, T)$ is the value at time t of a contract that pays par minus the recovery-given-default if the i 'th entity defaults between time 0 and time T . It is important to note that if the i 'th entity defaulted between 0 and t , then $FEP_t^i(0, T)$ is par minus recovery discounted on the time t Libor curve from t to T .

The value at time t of the default-adjusted forward portfolio swap is

$$V_t = \sum_{i=1}^N \frac{1}{N} V_t^i$$

where N is the number of reference entities. The factor $1/N$ is used because everywhere the notional of a contract is assumed to be par unless otherwise specified.

If the swaption is exercised, the protection buyer must pay the exercise price to the protection seller. The exercise price is specified by the strike spread, K , but also depends on the Libor curve at option maturity. For valuation purposes, we assume that *Libor rates are deterministic*. We can therefore use today's forward Libor curve at swaption maturity to calculate the exercise price. It is given by:

$$P(K) = \gamma(K)(K - FR) \tag{1}$$

where $\gamma(K)$ is the risky PV01 from T to T_M calculated on a credit curve that has been fitted to a flat CDS term structure with spreads equal to K, using a recovery-given-default of 40% (see section 2.2 for details).

Because the front-end protection values at T, $FEP_T^i(0,T)$, incorporate defaults between 0 and T, we can write the terminal swaption values as:

$$PS_T(K) = \max\{V_T - P(K), 0\} \tag{2}$$

$$RS_T(K) = \max\{P(K) - V_T, 0\} \tag{3}$$

where PS is the payer and RS is the receiver. We see that $PS_T(K) - RS_T(K) = V_T - P(K)$ and we get the put-call parity:

$$PS_0(K) - RS_0(K) = V_0 - P(K)D(0,T) \tag{4}$$

where $D(0,T)$ is the Libor discount factor from 0 to T.

When pricing the default-adjusted forward portfolio swap, it is important that the portfolio swap itself is priced correctly off the individual credit curves, ie, the intrinsic portfolio spread should equal the current market portfolio spread. In practice, this requires that the individual curves are adjusted as explained in section 2.1.1.

4.2. Pricing the payers and receivers

To price the swaptions, we propose a stochastic model for the value of the default-adjusted forward portfolio swap at swaption maturity, V_T , and thus for the terminal swaption values. We specify the distribution of V_T through what we call the default-adjusted forward portfolio spread, denoted X_T . For any time t between 0 and T, X_t is given by:

$$V_t = \gamma(X_t)(X_t - FR)D(t,T) \tag{5}$$

The function γ and the constant FR were defined in the previous section. $D(t,T)$ is the Libor discount factor from t to T.

When $K = X_0$ then $V_0 = \gamma(K)(K - FR)D(0,T) = P(K)D(0,T)$, and from the put-call parity $PS_0(K) = RS_0(K)$. In words: the default-adjusted forward portfolio spread is the strike for which the value of the payer is equal to the value of the receiver. Because of this relationship we can simply call the default-adjusted forward portfolio spread the *forward portfolio spread* when no defaults have yet occurred. So X_0 is the forward portfolio spread at time 0 for time T.

If no defaults have occurred at T, then the default-adjusted forward portfolio swap is the portfolio swap itself, and V_T is the value to the protection buyer of all the underlying CDS. By the definition of X_T ($V_T = \gamma(X_T)(X_T - FR)$), X_T is the portfolio spread at T. This is consistent with the above terminology of calling X_t the forward portfolio spread at t for T when no defaults have occurred before t.

When specifying the distribution of X_T , we must ensure that the default-adjusted forward portfolio swap is priced correctly in the model. X_T will be specified under the risk neutral measure (where $D(t,T)$ is the numeraire price at time $t \leq T$). Specifically, we must ensure that:

$$V_0 = D(0,T)E[V_T]$$

or equivalently, using (1) and (5):

$$E[P(X_T)] = P(X_0)$$

We assume that X_T is lognormally distributed with:

$$X_T = X_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)T + \sigma\sqrt{T}\varepsilon\right)$$

where ε is a standard normal random variable. μ is the drift and σ the volatility of the default adjusted forward portfolio spread. We choose this specification because of its simplicity and because a lognormal distribution is fairly consistent with empirical evidence. It is not difficult to imagine that the X_t process may contain jumps. X_t may jump, for example, if there is a surprise default of one of the reference entities. On the other hand, an entity usually defaults only when its spreads are already high, in which case the jump in the portfolio spread at the actually time of default may be small. In general, the lognormal distribution in our portfolio context seems more appropriate than assuming a lognormal spread for a single entity.

The drift μ is the free parameter used to ensure that $E[P(X_T)] = P(X_0)$. The function P is well behaved, and can be very well approximated using a polynomial spline. Even a simple second-order Taylor approximation of P around X_0 works well as long as the variance of X_T is not too high. In practice, the drift will be close to 0.

Once the drift has been fixed, and we have the approximation of the function P , it is straightforward to price the swaptions by discounting the expected terminal value using the distribution of X_T . Substituting (5) and (1) into (2) and (3), and using the fact that P is an increasing function, we get:

$$PS_0(K) = D(0, T)E\left[(P(X_T) - P(K))1_{(X_T \geq K)}\right]$$

$$RS_0(K) = D(0, T)E\left[(P(K) - P(X_T))1_{(K \geq X_T)}\right]$$

The equations can be solved in closed form when P is approximated by a polynomial spline.

4.3. Pricing examples

We illustrate by pricing CDX swaptions that mature on March 22, 2004. The underlying CDX swap matures on March 20, 2009. We did the valuation on November 6, 2003, at a time when the CDX spread was 56.

Figure 1. Adjusted CDS spreads (in bp) on the CDX reference entities on November 6, 2003

Maturity	6M	1Y	2Y	3Y	4Y	5Y	7Y
Average	35.6	39.0	44.1	48.8	52.3	55.6	61.3
Maximum	206	228	241	264	277	281	295
Median	22	24	29	33	38	40	46
Minimum	4	4	4	9	9	11	15

The first step in pricing the swaptions is to choose the best possible CDS spread curves for the 125 reference entities. Usually it suffices to base the curves on yesterday’s closing levels. The curves can then be adjusted to current market levels by using liquid CDS spreads (eg, parallel shifting based on the 5-year CDS spreads). After the individual curves have been adjusted to the current single-name market, all the curves must be adjusted so that the

portfolio of the 125 single-name CDS is priced according to the market CDX spread of 56, ie, so that the intrinsic CDX spread is 56. This adjustment also incorporates the fact that CDX is No-R.

To price the swaptions, we must choose a spread volatility. The spread volatility in our model is not the same as the spread volatility to be used with the Black formula. We model the default-adjusted forward portfolio spread which incorporates the defaults that occur before swaption maturity. The volatility in our model should therefore be higher than the volatility used in the Black formulas. Based on market prices, we choose a default-adjusted forward portfolio spread of 55%. Figure 2 shows the computed payer and receiver values. In parentheses next to a price is the implied Black volatility when the average adjusted CDS spread curve of the reference entities is used in the Black formula.

Figure 2. CDX swaption prices (in \$ per \$100 par) and implied Black volatilities off the average adjusted CDS spread curve of the reference entities

Strike (in bp)	45	50	55	60	65	70	75
Discounted exercise price	-0.68	-0.45	-0.23	0	0.23	0.45	0.67
Payer price (implied Black vol.)	0.79 (35%)	0.63 (40%)	0.49 (41%)	0.38 (41%)	0.29 (39%)	0.22 (37%)	0.16 (31%)
Receiver price (implied Black vol.)	0.08 (52%)	0.15 (52%)	0.24 (51%)	0.35 (50%)	0.48 (48%)	0.64 (47%)	0.80 (45%)

There are two main lessons to draw from Figure 2.

- For high strikes, implied Black volatilities for payer swaptions decrease as the strike increases.
- The implied Black volatilities are not the same for payers and receivers with the same strike.

Figure 2 shows that, according to our model, the implied Black volatility for the payer swaptions should decrease as the strike increases for high strikes. The effect is very noticeable, especially when comparing payers with strikes of 70 and 75. When the strike is 75, the value of the payer according to our model is very close the value of front-end protection priced off the adjusted average CDS curve (0.16 versus 0.14). When the strike is 80, the payer value is too low for the implied Black volatility to be defined.

We also see that the Black volatilities implied from our model are not the same for payers and receivers with the same strike. This shows that the default-adjusted forward portfolio swap is not priced correctly off the adjusted average CDS spread curve. According to the put-call parity the payer price minus the receiver price plus the discounted exercise price is equal to the value of the default-adjusted forward portfolio swap. In the example above, the default-adjusted forward portfolio swap has a value of 0.03. In the Black formulas, the value of the payer minus the value of the receiver does not depend on the volatility. If the discounted exercise price is subtracted from that difference, we arrive at the default-adjusted forward portfolio swap value priced off the adjusted average CDS curve. The value is 0.09, which is 0.06 too high according to the individual curves. It is this mispricing that gives the different implied Black volatilities for the payers and receivers.

5. SUMMARY

We presented the details of the CDX and TRAC-X swap and swaption contracts, and introduced market-consistent terminology to discuss their pricing. We argued that Black's formulas for single-name default swaptions are inappropriate for pricing portfolio swaptions, with the mispricing especially noticeable for deep out-of-the-money payer swaptions. We presented an alternative model that directly models the terminal swaption values through a default-adjusted forward portfolio spread. Our methodology ensures that a combined long-payer short-receiver position, which is insensitive to spread volatility, is priced in a manner consistent with the credit curves of the reference entities. Finally, we illustrated with numerical examples that the price differences between our model and the Black formulas are significant.

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